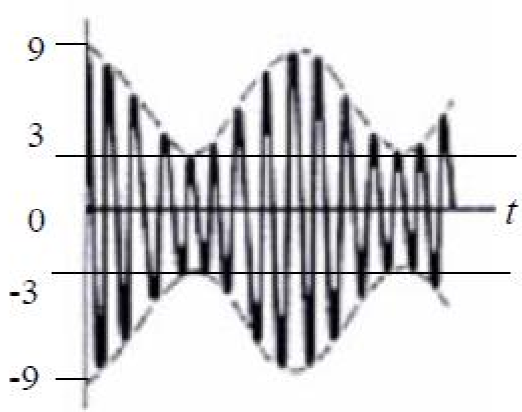
Solved Problems taken from:
http://course.ie.cuhk.edu.hk/~erg2310/
Courtesy of Prof. Chun-Kit CHAN

Problem 1
A sinusoidally modulated ordinary AM waveform is shown below.



(a) Determine the modulation index.
(b) Calculate the transmission efficiency.
(c) Determine the amplitude of the carrier which must be added to attain a modulation index of 0.3.

(a). Using $\begin{cases} s_{\max} = A_c(1+m_a) \\ s_{\min} = A_c(1-m_a) \end{cases}$ $\therefore\ A_c = 6$; the modulation index $m_a = \frac{1}{2}$

(b). $\mu = \dfrac{m_a^{\ 2}}{2+m_a^{\ 2}} = \dfrac{1}{9}$

(c). Using $m_a = \dfrac{a_m}{A_c}$, the new required carrier amplitude $A_c' = 10$

So, the amplitude of the carrier should be added by 4 to achieve the 0.3 modulation index.

Problem 2
The efficiency μ of a single-tone AM signal is defined as the percentage of the total power carried by the sidebands, that is:

$$\mu = \frac{P_{sig}}{P_t} \times 100\%$$

where P_{sig} is the power carried by the sidebands and P_t is the total power of the AM signal.
(a) Find *μ* for AM modulation index *ma*=0.5.
(b) Show that for a single-tone AM, *μmax* is 33.3% at *ma* =1.

$$P_c = \text{carrier power} = \frac{1}{2}A_c^2 \quad ; \; P_S = \text{sideband power} = \frac{1}{4}m_a^2 A^2$$

$$\text{Thus,} \quad \mu = \frac{P_S}{P_t} = \frac{m_a^2}{2 + m_a^2}$$

(a). For m_a=0.5, $\mu = 11.1\%$

(b). $\mu_{\max}$ occurs at m_a=1, ➔ y $\mu_{\max} = 33.3\%$

Problem 3
The output signal from an AM modulator is:

$s(t)$ = 5cos(1800π t) + 20cos (2000π t) + 5cos (2200π t)

(a) Determine the modulation index.
(b) Determine the ratio of the power in the sidebands to the power in the carrier.

(a)

$$s(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$
$$= 20\left[1 + \frac{1}{2}\cos(200\pi)\right]\cos(2000\pi t)$$

Thus, modulation index is 0.5

(b) Carrier power= 200
Sideband power = 25
Sideband power to carrier power ratio = 0.125

Problem 4

Assume that the transmitter circuitry limits the modulated output signal to a certain peak voltage value, say ***X*** volts. By employing this transmitter, show that the sideband power of a DSB-SC signal with a peak voltage of ***X*** is four times that of an AM (DSB-LC) signal having the same peak voltage and 100% modulation index, and is half that of a SSB-SC signal having the same peak voltage.

(1) DSB-SC: $s(t) = Ax(t)\cos(2\pi f_c t)$, peak voltage = A ➔ $A = X$

➔ $P_{sb} = \dfrac{X^2}{4} P_x$

(2) DSB-LC: $s(t) = A[1 + x(t)]\cos(2\pi f_c t)$, peak voltage = $2A$ ➔ $A = X/2$

➔ $P_{sb} = \dfrac{X^2}{16} P_x$

(3) SSB-SC: $s(t) = Ax(t)\cos(2\pi f_c t) - A\hat{x}(t)\sin(2\pi f_c t)$

peak voltage = $A\sqrt{\overline{x^2(t)} + \overline{\hat{x}^2(t)}}$ ➔ $A = X / \sqrt{\overline{x^2(t)} + \overline{\hat{x}^2(t)}}$

$$P_{sb} = \begin{cases} \dfrac{X^2}{2} P_x & \text{for} \quad \sqrt{\overline{x^2(t)} + \overline{\hat{x}^2(t)}} = \sqrt{2} \\ X^2 P_x & \text{for} \quad \sqrt{\overline{x^2(t)} + \overline{\hat{x}^2(t)}} = 1 \end{cases}$$

Hence, the sideband power of a DSB-SC signal is four times that of an AM (DSB-LC) signal, and is half (or one-quarter) that of a SSB-SC signal.

Problem 5

A single-tone modulating wave $m(t) = A_m\cos(2\pi f_m t)$ is used to generate the VSB modulated wave:

$$s(t) = \alpha A_m A_c \cos[2\pi (f_c + f_m) t] + A_m A_c (1-\alpha)\cos[2\pi (f_c - f_m) t],$$

where α is a constant ($\alpha \le 1$), A_c is the amplitude of carrier, and f_c is the frequency of carrier.

(a) Express $s(t)$ in the form of $I(t)\cos(2\pi f_c t) + Q(t)\sin(2\pi f_c t)$, where $I(t)$ and $Q(t)$ are called the in-phase and quadrature components.

(b) What is the value of the constant α for which $s(t)$ reduces to a DSB-SC modulated wave?

(c) What are the values of the constant α for which $s(t)$ reduces to a SSB modulated wave?

(d) The VSB wave $s(t)$, plus a carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced.

(a). Expanding $s(t)$, we get

$$s(t) = A_m A_c \cos(2\pi f_m t)\cos(2\pi f_c t) + A_m A_c (1-2\alpha)\sin(2\pi f_m t)\sin(2\pi f_c t)$$

(b). Because $s(t) = A_m A_c \cos(2\pi f_c t)\cos(2\pi f_m t) + A_m A_c (1-2\alpha)\sin(2\pi f_c t)\sin(2\pi f_m t)$

When $\alpha = \frac{1}{2}$, the VSB is reduced to DSB-SC.

(c). When $\alpha = 1$ or $\alpha = 0$, the VSB is reduced to SSB-SC.

(d) the output from envelope detector:

$$r(t) = A_c[1 + A_m \cos(2\pi f_m t)]\sqrt{1 + \left[\frac{A_m(1-2\alpha)\sin(2\pi f_m t)}{1 + A_m \cos(2\pi f_m t)}\right]^2}$$

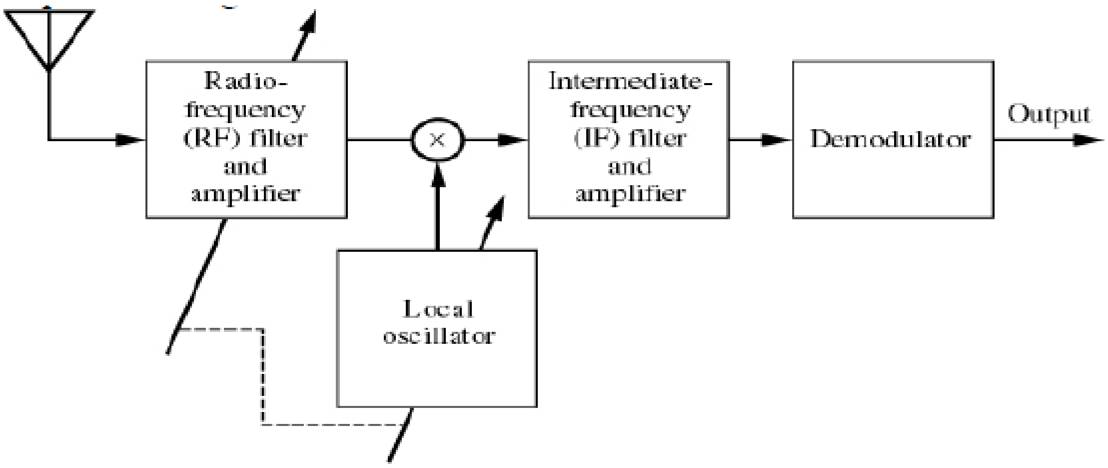
$$\Rightarrow d(t) = \sqrt{1 + \left[\frac{A_m(1-2\alpha)\sin(2\pi f_m t)}{1 + A_m \cos(2\pi f_m t)}\right]^2}$$

Problem 6

A radio receiver used in the AM system is shown below. The mixer translates the carrier frequency f_c to a fixed IF of 455kHz by using a local oscillator of frequency f_{LO}. The broadcast-band frequencies range from 540kHz to 1600kHz.

(a) Determine the range of tuning that must be provided in the local oscillator (i) when f_{LO} is higher than f_c (superheterodyne receiver) and (ii) when f_{LO} is lower than f_c.

(b) Based on the results obtained in (a), explain why the usual AM radio receiver uses a superheterodyne system.



(a). When $f_{LO} > f_C$, the required tuning range of the local oscillator is 995kHz - 2055kHz.

When $f_{LO} < f_C$, the required tuning range of the local oscillator is 85kHz - 1145kHz.

(b) The frequency ratio, that is, the ratio of the highest f_{LO} to the lowest f_{LO}, is 2.07 for $f_{LO} > f_C$ and 13.47 for $f_{LO} < f_C$. ➔ $f_{LO} > f_C$ (superheterodyne) is preferred.

Problem 7

A particular version of AM stereo uses quadrature multiplexing. Specifically, the carrier $A_c\cos(2\pi f_c t)$ is used to modulate the sum signal $m_1(t) = V_o + m_L(t) + m_R(t)$ where V_o is a DC offset included for the purpose of transmitting the carrier component, $m_L(t)$ is the left-hand audio signal, and $m_R(t)$ is the right-hand audio signal. The quadrature carrier $A_c\sin(2\pi f_c t)$ is used to modulate the difference signal $m_2(t) = m_L(t) - m_R(t)$

(a) Show that an envelope detector may be used to recover the sum $m_L(t) + m_R(t)$ from the quadrature-multiplexed signal. How would you minimize the signal distortion produced by the envelope detector?

(b) Show that a synchronous/coherent detector can recover the difference signal, $m_L(t) - m_R(t)$.

(c) How are the desired $m_L(t)$ and $m_R(t)$ finally obtained?

The transmitted signal is:

$$s(t) = A_c[V_0 + m_L(t) + m_R(t)]\cos(2\pi f_c t) + A_c[m_L(t) - m_R(t)]\sin(2\pi f_c t)$$

(a) The envelope detection output: $r(t) = A_c(V_0 + m_L(t) + m_R(t))\sqrt{1 + \left(\frac{m_L(t) - m_R(t)}{V_0 + m_L(t) + m_R(t)}\right)^2}$

If we choose a large value of V_0 ➔ $r(t) \approx A_c(V_0 + m_L(t) + m_R(t))$

(b). Multiply *s(t)* by $A_c \sin(2\pi f_c t)$ and followed by a low-pass filter, $[m_L(t) - m_R(t)]$ is obtained.

(c). With the known values of $[m_L(t) + m_R(t)]$ and $[m_L(t) - m_R(t)]$, it is easy to get $m_L(t)$ and $m_R(t)$.

Problem 8

A normalized sinusoidal signal ***a(t)*** has a bandwidth of 5,000 Hz and its average power is 0.5W. The carrier *A*cos2π*fct* has an average power of 50W. Determine the bandwidth and the average power of the modulated signal if the following analog modulation scheme is employed:

(a) single-side band modulation with suppressed carrier modulation (SSB-SC), which is generated by phase-shift method with the given carrier ;

(b) double-side band with suppressed carrier modulation (DSB-SC) ;

(c) AM or double-side band with large carrier (DSB-LC) with a modulation index of 0.8.

As the average power of the carrier is:

$P_c = \frac{A^2}{2} = 50,$ **so the A=10.**

a) SSB-SC:

$$s(t) = Aa(t)\cos(2\pi f_c t) - A\hat{a}(t)\sin(2\pi f_c t)$$

$$P = \overline{s^2(t)} = \frac{1}{2}A^2 \times \overline{a^2(t)} + \frac{1}{2}A^2 \times \overline{\hat{a}^2(t)} + 0 = A^2 \times P_a = 100 \times 0.5 = 50\,\text{w}$$

Bandwidth is 5000 Hz.

b) DSB-SC:

$$s(t) = Aa(t)\cos(2\pi f_c t)$$

$$P = \overline{s^2(t)} = \frac{1}{2}A^2 \times \overline{a^2(t)} = 50*0.5 = 25\text{w}$$

Bandwidth is 10000 Hz.

c) AM or DSB-LC:

$$s(t) = A[1 + 0.8a(t)]\cos(2\pi f_c t)$$

$$P = \overline{s^2(t)} = \frac{1}{2}A^2[1 + 0.8^2 \times \overline{a^2(t)}] = \frac{1}{2}A^2[1 + 0.8^2 \times P_a] = 50*[1 + 0.64*0.5] = 66\,\text{w}$$

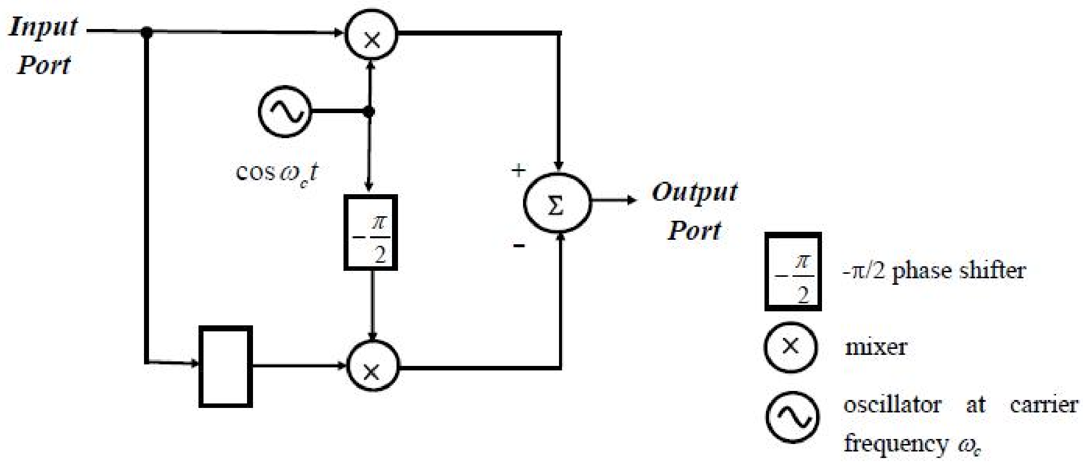
Bandwidth is 10000 Hz.

Problem 9

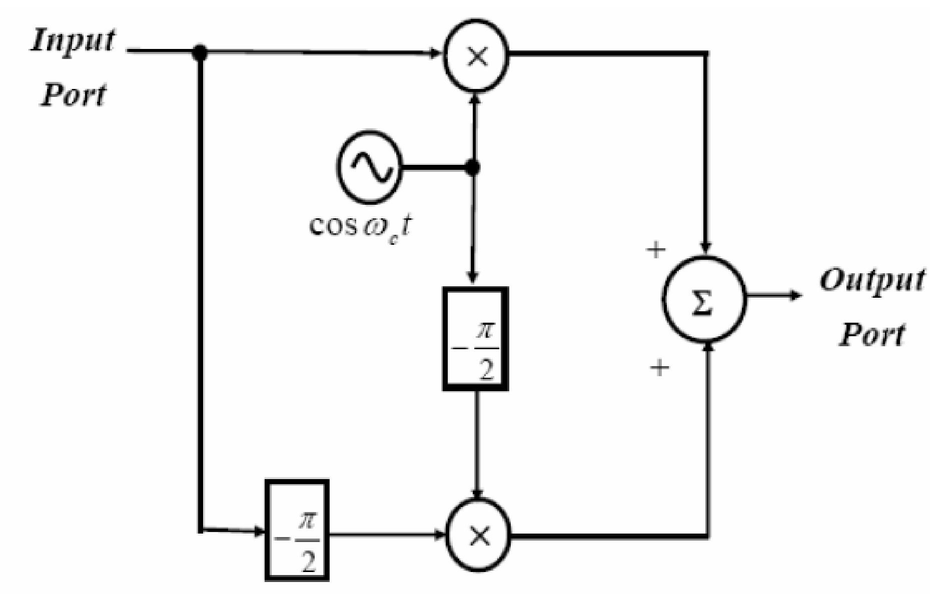
Consider the following circuit:

(i) An upper-sideband single-sideband (SSB) can be generated by feeding a message signal ***m(t)*** into the input port of the above circuit. This is known as phase-shifted method for SSB signal generation. Determine the modulated signal ***s(t)*** at the output port.

(ii) Suggest one appropriate modification of the above circuit so that the modified circuit can be used to demodulate the upper-sideband SSB signal obtained in part (b)(i) and retrieve the message signal *m(t)*. Prove and verify your modified circuit as an upper-sideband SSB signal demodulator.



Soln:



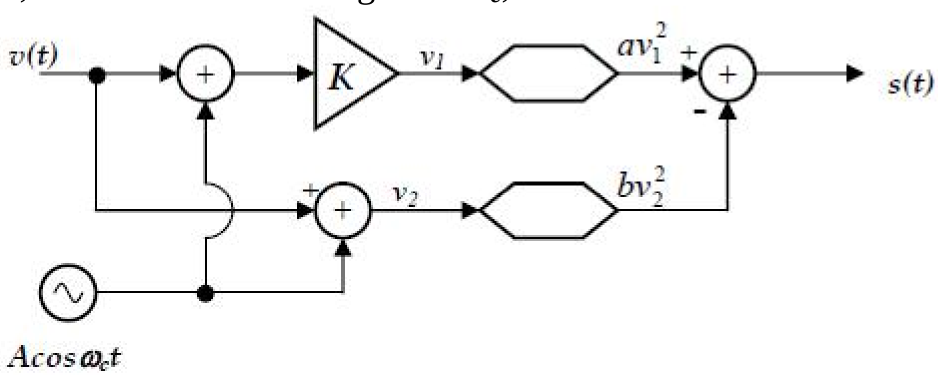
Proof:

$$\hat{s}(t) = m(t)\cos(\omega_c t - \frac{\pi}{2}) - \hat{m}(t)\sin(\omega_c t - \frac{\pi}{2})$$
$$= m(t)\sin(\omega_c t) + \hat{m}(t)\cos(\omega_c t)$$

$$y(t) = s(t)\cos\omega_c t + \hat{s}(t)\sin\omega_c t$$
$$= \left(m(t)\cos\omega_c t\cos\omega_c t - \hat{m}(t)\sin\omega_c t\cos\omega_c t\right)$$
$$+ \left(m(t)\sin\omega_c t\sin\omega_c t + \hat{m}(t)\cos\omega_c t\sin\omega_c t\right)$$
$$= m(t)\left(\cos^2\omega_c t + \sin^2\omega_c t\right)$$
$$= m(t)$$

Problem 10

Consider the following circuit with an input signal, ***v(t)**, an amplifier with gain **K***, a sinusoidal carrier signal at ω_c, and *two nonlinear devices:*



(a) Express the output signal ***s(t)*** in terms of ***v(t)***, ***A***, ***K***, ***cos*** **(**ω_c***t*)**, ***a*** and ***b***.
(b) Determine the choice of the gain ***K*** so that the above circuit performs as a DSB-SC modulator without output filtering.

(a)

$$s(t)$$
$$= a*[k(v + A\cos(w_c t))]^2 - b[v - A\cos(w_c t)]^2$$
$$= (ak^2 - b)v^2 + 2Av(ak^2 + b)\cos w_c t + (ak^2 - b)A^2 \text{co}$$

(b) $k = \pm\sqrt{\dfrac{b}{a}}$. **(assume ab>0)**

(Just to make $ak^2 - b = 0$ is enough)

Remark 1: I noticed some students also require $2Av(ak^2 + b)$=A to hold in their solutions because they thought the standard form of DSB-LC is $Av(t)\cos w_c t$ and they just compare the two. That is not correct.

I want to point out that the "A" is just a constant, not definitely the same as the "A" in the standard form. It's OK only if it is a constant. The main spirit is to tell you it is constant.

Remark 2: K can be negative. I noticed some students ignore $k = -\sqrt{\dfrac{b}{a}}$ because it is negative and not suitable for amplifier.

For example, see the operator amplifier below,